COUPLED DYNAMIC ANALYSIS OF MOORING LINES FOR DEEP WATER FLOATING SYSTEMS

K. Gurumurthy\textsuperscript{1,2}, S. Ahmad\textsuperscript{2} and A.S. Chitrapu\textsuperscript{3}

Abstract: As the floating systems extend to deeper waters, damping induced by mooring lines increases, relative to other sources of damping, and the dynamic response of mooring lines become predominantly dependent on the dynamic behavior of the whole floating system. Hence the dynamic interaction effects between the floater and its mooring system must be considered to predict the response of mooring lines more accurately. In this paper, coupled dynamic analysis of mooring lines for a deep water classical spar floating system is carried out in random waves and current and the results are compared with quasi-static and decoupled dynamic analyses. From the results, it is found that mooring line damping reduces the standard deviation of slow-drift surge amplitude by as much as about 14\% and standard deviation of low frequency mooring line tension by about 4\%. In the wave frequency range, mooring line tension predicted by coupled dynamic analysis can be eight times greater than the tension predicted by quasi-static analysis. It is seen that mooring line tension increases in presence of currents. The coupled dynamic analysis method captures all significant coupling effects and enhances the accuracy of the predicted dynamic response of mooring lines for deep water floating systems.

Keywords: deep water; floating systems; spar; mooring lines; quasi-static; decoupled; coupled.

INTRODUCTION

Depletion of reserves in shallow water depths has motivated the oil and gas industry to move towards deep water resulting in floating rather than fixed offshore structures. The floating system comprises the floater, the mooring lines, and all risers connected to the floater. Design of the mooring lines and risers are dominated by the motions of the floater. An over prediction of motion would require costly risers and moorings, whilst an under prediction can lead to inadequate designs with possible catastrophic consequences. Accurate prediction of motions of a floater is very important for the integrity and associated costs of the risers, and moorings. As the water depth increases, the damping induced by mooring lines increase.

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relative to other sources of damping, affecting the low frequency motion response of the floater considerably. Besides this, the mooring line response can be strongly dependent on the dynamic behavior of the whole floating system.

Traditionally, the mooring line analysis is undertaken in two separate stages. The first stage involves the computation of floater motions treating the floater dynamically, wherein the restoring forces are modeled quasi-statically as nonlinear springs. The second stage is a dynamic analysis of the moorings and risers, applying the floater motions as boundary conditions at the top ends. This is a de-coupled analysis due to the fact that the dynamics of mooring system are calculated separately from the dynamics of the floater. In deep water, mooring lines and risers becomes longer and heavier and the resulting inertia and damping effects from them are important. Coupled dynamic analysis which includes the risers, moorings and floaters in a single model captures all dynamic interaction effects. The slow drift damping due to the risers and moorings is automatically accounted. Thus, the coupled dynamic analysis predicts the floater motions and mooring line tensions more accurately.

In this paper, coupled dynamic analysis of mooring lines for deep water classical spar floating system has been carried out in random waves and collinear uniform current in a water depth of 1018 m. The mooring line fairlead tensions are compared with the quasi-static and de-coupled dynamic analyses. The spar is modeled as a rigid body with six degrees-of-freedom and the mooring lines are modeled using a finite element (FE) representation of an elastic rod. The coupling between the spar and mooring system is achieved by imposing hinge connection at the fairleads. At every time step of the integration of equations of motion of the spar, a series of nonlinear dynamic analysis of the mooring lines is performed using a subcycling technique. Wave kinematics are computed using linear (Airy) wave theory with Wheeler stretching method. The Morison equation has been used for force computation. The coupled dynamic analysis model can consider several non-linear effects in floater and mooring line dynamics and the dynamic interaction between the floater and its mooring lines. Section 2 outlines the coupled dynamic analysis methodology. This methodology is applied to a large diameter classical spar-mooring system whose details and data are given section 3. The results and discussion are presented in section 4.

COUPLED DYNAMIC ANALYSIS

The analysis procedure is based on the time domain numerical integration of equation of motion of floater hull and mooring lines. For coupled dynamic analysis the motion equations of the hull and dynamic equations of a mooring system are solved simultaneously. The motion equations for a floater hull, mooring line and the numerical schemes for integration of the coupled dynamic equations were detailed in Gurumurthy et al. (2008). A brief outline of the coupled dynamic analysis methodology is presented here.

Equations of Motion of a Floater
The equations of motion of a rigid body, are given in terms of the external forces and moments and the mass and inertia properties of the body, (see, for example, Chitrapu et al., 1998).
\[ \frac{d\mathbf{\ddot{v}}}{dt} = [M]^{-1} \mathbf{\ddot{F}} ; \quad \frac{d\mathbf{\ddot{x}}}{dt} = \mathbf{\ddot{v}} , \]
\[ \frac{d\mathbf{\ddot{\omega}}}{dt} = [I]^{-1} \{ \dot{\mathbf{M}}_b - \mathbf{\ddot{\omega}} \times ([I] \mathbf{\ddot{\omega}}) \}; \quad \frac{d\mathbf{\ddot{\theta}}}{dt} = [B]^{-1} \mathbf{\ddot{\omega}} \]

where \( \mathbf{x}, \mathbf{v} \) are the translational displacement and velocity vectors of the origin of the body-fixed coordinate system, \( \mathbf{\dot{\omega}}, \mathbf{\ddot{\omega}} \) are the angular displacement and velocity vectors, \( \mathbf{\ddot{F}} \) is the total external force on the floater in a space fixed system and it includes hydrostatic (buoyancy) force, Froude-Krylov force, force due to body- and wave- particle accelerations, relative velocity (viscous drag) force and force due to the mooring system. Other forces such as due to wind can be added to this force. \( \mathbf{M}_b \) is the total external moment about the origin of the body-fixed system. \([M], [I]\) are body mass and inertia matrices and \([B]\) is the transformation matrix relating angular velocity to time-derivatives of the Euler angles. All the above forces are evaluated over the instantaneous submerged length of the floater in its displaced position.

Equation (1) represents a system of 12 first-order, coupled, nonlinear ordinary differential equations, with \( \mathbf{x}, \mathbf{v}, \mathbf{\dot{\omega}} \) and \( \mathbf{\ddot{\omega}} \) being the 12 state variables, and these are solved using a fourth order Runge-Kutta (R-K) time integration method.

Equations of Motion for a Mooring Line
The governing equation for mooring line motion is based on the theory of dynamics of slender rods first introduced by Garrett (1982) and expanded by Paulling and Webster (1986) to include stretch and various loads appropriate to the line dynamic problem. These include the effect of gravity forces due to the mass of the line, buoyancy and hydrodynamic forces due to the mooring line and wave motion. The ocean bottom is assumed flat and elastic (see Webster, 1995).

Assuming that the instantaneous configuration of an elastic rod can be expressed as a vector distance, \( \mathbf{r} \), the governing equation of motion for the mooring line can be given as follows (see Paulling and Webster, 1986).
\[
m \dddot{r} + (B \dot{r}^2 - \lambda \dot{\kappa}) = q
\]
with a stretch constraint equation
\[r' \dot{r}' = (1 + \varepsilon)^2
\]
Where \( B \) is the bending stiffness; \( \lambda = T - Bk^2 \), \( T \) is the tension and \( k \) is the local curvature. \( q \) is the external force per unit length, \( m \) is the mass per unit length, \( \varepsilon = T/AE \) and \( AE \) is axial stiffness of the rod.

Equations (2) and (3) can be solved using a nonlinear dynamic analysis of the Finite Element Model (FEM) of each line (see Garrett, 1982, Paulling and Webster, 1986). A FE solution method was employed to discretize the above vector governing equations into algebraic
equations. Based on Galerkin’s approach, a set of shape functions are used to approximate the mooring line and the variations of tensions along the length. By using this method, the original vector equations are converted into 15 scalar equations with 15 unknowns which are solved using second-order Adams-Moulton integration algorithm.

Coupling of Floater Hull and Mooring Lines
Motion equations of hull and dynamic equations of its mooring system are coupled by imposing ideal hinges at their fairleads. The right hand side of equation (1) includes forces and moments \( \mathbf{F} \) and \( \mathbf{M} \), acting on the floater, including those resulting from environmental loadings and mooring system. The mooring lines are analyzed under the action of all environmental loadings and also with the displacements of hull motion prescribed at the top of each line. The dynamic boundary conditions are implemented by letting the total tension at the top end of all mooring lines at the upper end be equal to their force on the floater hull. This is the key to the scheme considered for coupling between the hydrodynamic model of the hull and the FE structural model of the mooring lines.

In the simulation of the coupled dynamic response of a floater-mooring system it is necessary to integrate all the equations of motion simultaneously, one set for the hull and a separate set for each mooring line. The requirement of the Runge-Kutta algorithm, used for hull motion predictions, for two additional estimates of derivatives in the mid-time interval is incompatible with the mooring line integration scheme. To overcome this difficulty, Paulling and Webster (1986) adopted Hammings algorithm, a fourth-order predictor-corrector scheme for the hull motions. In this paper a domain decomposition method called subcycling technique is employed that takes into account the natural partition that exists between the hull and the mooring lines (see Rodrigues et al., 2006).

APPLICATION

The methodology presented above has been applied to determine the dynamic response of mooring lines for a large diameter classical spar floating system, which was studied in a Joint Industry Project (JIP). The mooring system consists of four catenary mooring lines. The particulars of this spar and mooring line properties are given in Table 1. The characteristics of mooring lines and hydrodynamic coefficients considered by Chen et al. (2001) are chosen for this study. The model of the spar – mooring system is shown in Figure. 1. The numerically simulated tension-offset curves of lines 1 and 3 are shown in Figure. 2 and surge static offset curve of the whole mooring system is shown in Figure. 3.

A uniform added mass coefficient of 1.0, a surge drag coefficient of 0.6 and a heave drag coefficient of 3.0 have been used for the spar. The added mass and drag coefficients of mooring lines are set to be 1.0 and 1.2 respectively.
Table 1. Main particulars of JIP Spar and mooring line properties

<table>
<thead>
<tr>
<th></th>
<th>JIP Spar</th>
<th>Mooring line</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>40.54 m</td>
<td>Number of mooring lines 4</td>
</tr>
<tr>
<td>Draft</td>
<td>198.12 m</td>
<td>Length of mooring line 2000 m</td>
</tr>
<tr>
<td>Mass (with entrapped water)</td>
<td>2.592 * 10^8 Kg</td>
<td>Mass per unit length 1100 kg/m</td>
</tr>
<tr>
<td>Buoyancy</td>
<td>2.569 * 10^9 N</td>
<td>Elastic stiffness (EA) 0.15e10 N</td>
</tr>
<tr>
<td>Radius of gyration (Pitch)</td>
<td>62.33 m</td>
<td>Pretension 1.37e07 N</td>
</tr>
<tr>
<td>Center of gravity (from still water line)</td>
<td>-105.98 m</td>
<td>Mooring Point (from still water line) -106.62 m</td>
</tr>
<tr>
<td>Water depth</td>
<td>1018 m</td>
<td>Total weight in water (% spar weight) 2.96%</td>
</tr>
</tbody>
</table>

Fig. 1. Model of the spar - mooring System
RESULTS AND DISCUSSION

The response of mooring lines for spar floating system in unidirectional random waves and collinear uniform current is calculated using a coupled dynamic analysis and the results are compared with quasi-static and de-coupled dynamic analyses in a water depth of 1018 m. In coupled dynamic analysis, first the static equilibrium position of the spar – mooring system is determined and the time domain simulation for the coupled system is performed from this position. In de-coupled dynamic analysis, floater motions from quasi-static analysis are applied at the top ends of mooring lines. It should be noted that in de-coupled dynamic analysis, in general, additional coupling contributions from damping as well as current loading on the mooring system need to be assessed and given as input to the quasi-static analysis to improve the final result. However, no such contributions are considered here.

In the simulation the time step used for the spar hull is 0.1 s and for lines is 0.025 s and length of the simulation is 3 hrs. A 400 s cosine taper function is used at the beginning of the analysis to take care of the transient response. Each mooring line was modeled with 20 quadratic elements of uniform size. The response of spar in all modes and the tensions of all mooring lines are predicted, but only the simulated C.G. surge response of spar and mooring line 1 fairlead tensions are presented here. The statistics of the surge motion and tensions in mooring line 1 are summarized, where the contributions of low-frequency (LF, 0-0.15 rad/sec) and wave-frequency (WF, 0.151-1.2 rad/sec) are separated after removing the mean.

Random Waves without Current
For random wave simulations, the JONSWAP spectrum with significant wave height of 13.1 m and peak period of 14 s with a peakedness parameter of 2 is used. Airy wave theory with Wheeler stretching has been used for particle kinematics. The time series of predicted surge response of spar simulated by the quasi-static and coupled dynamic analyses is shown in Figure. 4. The time series of predicted mooring line 1 tension simulated by the quasi-static, de-coupled dynamic and coupled dynamic analyses are shown in Figure. 5.
The statistics of the surge motion and mooring line 1 tensions are summarized in Tables 2 and 3.

From Table 2, the LF standard deviation of the surge simulated by coupled dynamic analysis is 14.0% smaller than quasi-static analysis. The decrease of LF surge predicted by coupled dynamic analysis is because of the damping of mooring lines. In the WF range the standard deviation of the surge simulated by coupled dynamic and quasi-static analysis is nearly same.
Table 2. Statistics of the surge of the spar in random waves without current

<table>
<thead>
<tr>
<th>Analysis Type</th>
<th>Mean (m)</th>
<th>WF std. dev. (m)</th>
<th>LF std. dev. (m)</th>
<th>Total std. dev. (m)</th>
<th>Extreme</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quasi-static</td>
<td>-0.46</td>
<td>0.72</td>
<td>2.10</td>
<td>2.21</td>
<td>6.31</td>
</tr>
<tr>
<td>Coupled dynamic</td>
<td>-0.59</td>
<td>0.72</td>
<td>1.80</td>
<td>1.93</td>
<td>5.31</td>
</tr>
</tbody>
</table>

Table 3. Statistics of mooring line 1 tension in random waves without current

<table>
<thead>
<tr>
<th>Analysis Type</th>
<th>Mean (KN)</th>
<th>WF std. dev. (KN)</th>
<th>LF std. dev. (KN)</th>
<th>Total std. dev. (KN)</th>
<th>Extreme (KN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quasi-static</td>
<td>13660</td>
<td>62.67</td>
<td>77.93</td>
<td>98.87</td>
<td>13998 13375</td>
</tr>
<tr>
<td>De-coupled dynamic</td>
<td>13660</td>
<td>487.20</td>
<td>171.90</td>
<td>511.90</td>
<td>15447 11939</td>
</tr>
<tr>
<td>Coupled dynamic</td>
<td>13660</td>
<td>480.60</td>
<td>165.50</td>
<td>504.30</td>
<td>15445 11986</td>
</tr>
</tbody>
</table>

From Table 3, in the WF range the ratio of standard deviation of the predicted tension in line 1 by coupled dynamic to quasi-static and de-coupled dynamic to quasi-static are 7.7 and 7.8 respectively. Thus the increase in the tension predicted by coupled dynamic or de-coupled dynamic analyses in WF range indicates that the dynamic forces in the mooring lines become more significant. In the LF range, the dynamic effects on the mooring lines are not very significant. However, the standard deviation of the tension simulated by coupled analysis is 3.72 % smaller than de-coupled analysis. The decrease of LF tension predicted by coupled dynamic analysis is because of the reduction of LF surge response.

Random Waves with Uniform Current
A uniform current profile with velocity of 1.3 m/s is assumed to act collinear with the same random wave conditions as mentioned in the section 4.1. It should be noted that the profile is quite unrealistic (uniform and constant over the depth, also wave and current headings are not collinear in most design cases). However, the uniform current profile is considered to study the effect of current loading on mooring lines. The response of mooring lines in unidirectional random waves and uniform collinear current is calculated using quasi-static, de-coupled and coupled dynamic analyses, but only the time series of predicted surge response of spar and mooring line 1 tension simulated by coupled dynamic analysis are shown in Figure. 6.
The statistics of the surge motion and mooring line 1 tension simulated by the quasi-static, de-coupled and coupled dynamic analyses are summarized in Tables 4 and 5.

**Table 4. Statistics of the surge of the spar in random waves with current**

<table>
<thead>
<tr>
<th>Analysis Type</th>
<th>Mean (m)</th>
<th>WF std. dev. (m)</th>
<th>LF std. dev. (m)</th>
<th>Total std. dev. (m)</th>
<th>Extreme</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quasi-static</td>
<td>59.29</td>
<td>0.71</td>
<td>0.86</td>
<td>1.10</td>
<td>Max 64.89</td>
</tr>
<tr>
<td>Coupled dynamic</td>
<td>70.75</td>
<td>0.71</td>
<td>0.74</td>
<td>1.01</td>
<td>Max 75.45</td>
</tr>
</tbody>
</table>

**Table 5. Statistics of mooring line 1 tension in random waves with current**

<table>
<thead>
<tr>
<th>Analysis Type</th>
<th>Mean (KN)</th>
<th>WF std. dev. (KN)</th>
<th>LF std. dev. (KN)</th>
<th>Total std. dev. (KN)</th>
<th>Extreme</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quasi-static</td>
<td>15930</td>
<td>78.50</td>
<td>64.00</td>
<td>99.35</td>
<td>Max 16354</td>
</tr>
<tr>
<td>De-coupled dynamic</td>
<td>16200</td>
<td>618.60</td>
<td>224.90</td>
<td>644.00</td>
<td>Max 18880</td>
</tr>
<tr>
<td>Coupled dynamic</td>
<td>16750</td>
<td>641.70</td>
<td>231.80</td>
<td>666.70</td>
<td>Max 19562</td>
</tr>
</tbody>
</table>

From Tables 2 and 4, it is seen that mean surge offset has increased due to current. In the LF range, the standard deviation of the surge response is appreciably reduced due to the currents, while the wave-frequency response almost remains the same. The reduction of LF surge is due to the increase in viscous damping by currents. From Tables 3 and 5, it is seen that the mooring line tensions have increased in the presence of currents. The increase of mooring line tension is due to the increase of mean surge.

From Tables 5, it is seen that the surge response and mooring line tensions are of similar trends as seen in the case without current except the mean tension. The mean tension
predicted by de-coupled or coupled dynamic analysis is more than the quasi-static analysis. This is due to additional current loads on mooring lines considered in dynamic analysis. The mooring line tension predicted by coupled analysis is more than de-coupled analysis. This is due to the fact that in strong current environments, current induced drag on the mooring lines changes the restoring force characteristics significantly.

CONCLUSIONS

Coupled dynamic analysis of mooring lines for a deep water classical spar floating system under random waves and current is carried out and the results are compared with quasi-static and de-coupled dynamic analyses. Coupled dynamic analysis model can consider several non-linear effects and the dynamic interaction between the spar and its mooring lines. Based on comparisons made between the results obtained from quasi–static and coupled dynamic analyses, it is seen that mooring line damping reduces LF surge response of the spar platform by about 14% in amplitude and standard deviation of low frequency mooring line tension by about 4%. Dynamic tensions in the mooring lines predicted by coupled or de-coupled dynamic analysis results in much greater tensions in the WF range and can be about eight times greater than the tension predicted by quasi-static analysis. It is also seen that mooring line tension increases in presence of currents. Coupled dynamic analysis method captures all significant coupling effects and can be used for accurate prediction of dynamic response of mooring lines for deep water floating systems. It should, however, be noted that the conclusions are based on analyses of spar floating system and are in general applicable for such systems only. Coupling effects in terms of damping and current loading due to slender structures in general depend on the type floater, water depth, mooring and riser system as well as excitation level.

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